Ice Sheet Models

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Introduction

- Ice flow modelling is a ***huge*** topic
  - Highly complex physical system
  - Many components interacting together
  - Maths is challenging
  - Multiple methods for approaching each part
  - Modelling is inadequately constrained by observation
  - Challenging spatial and temporal scales
- This talk gives an overview of ***a few*** important aspects

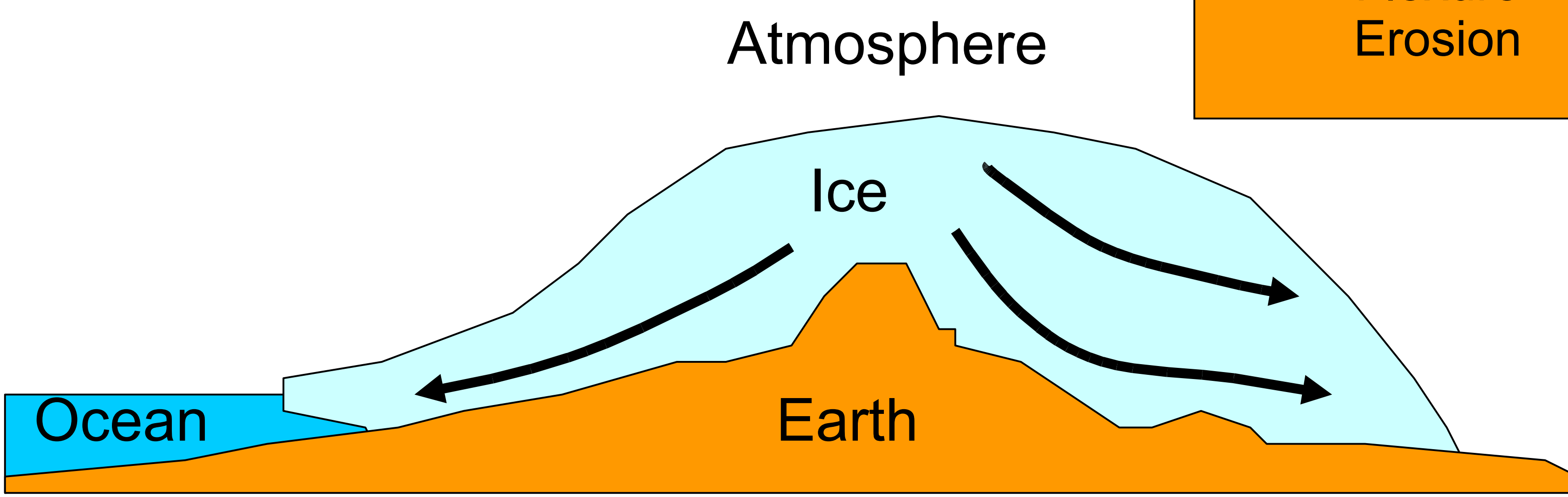
Ice sheet model components

Atmospeheric Boundary
Albedo/ Temperature
Precipitation / Circulation

ICE SHEET
Internal dynamics

Marine Boundary
Sea Level
Temperature

Basal Boundary
Friction
Flexure
Erosion



Modelling hierarchy

- Conceptual/analytical models
  - Simple physics (e.g. ice treated as plastic)
  - Can be solved analytically
  - Can yield insights into feedbacks, hysteresis, statistical properties
- Further reading:
  - Hans Oerlemans 'Simple Glacier Models' (from his website)

Modelling hierarchy

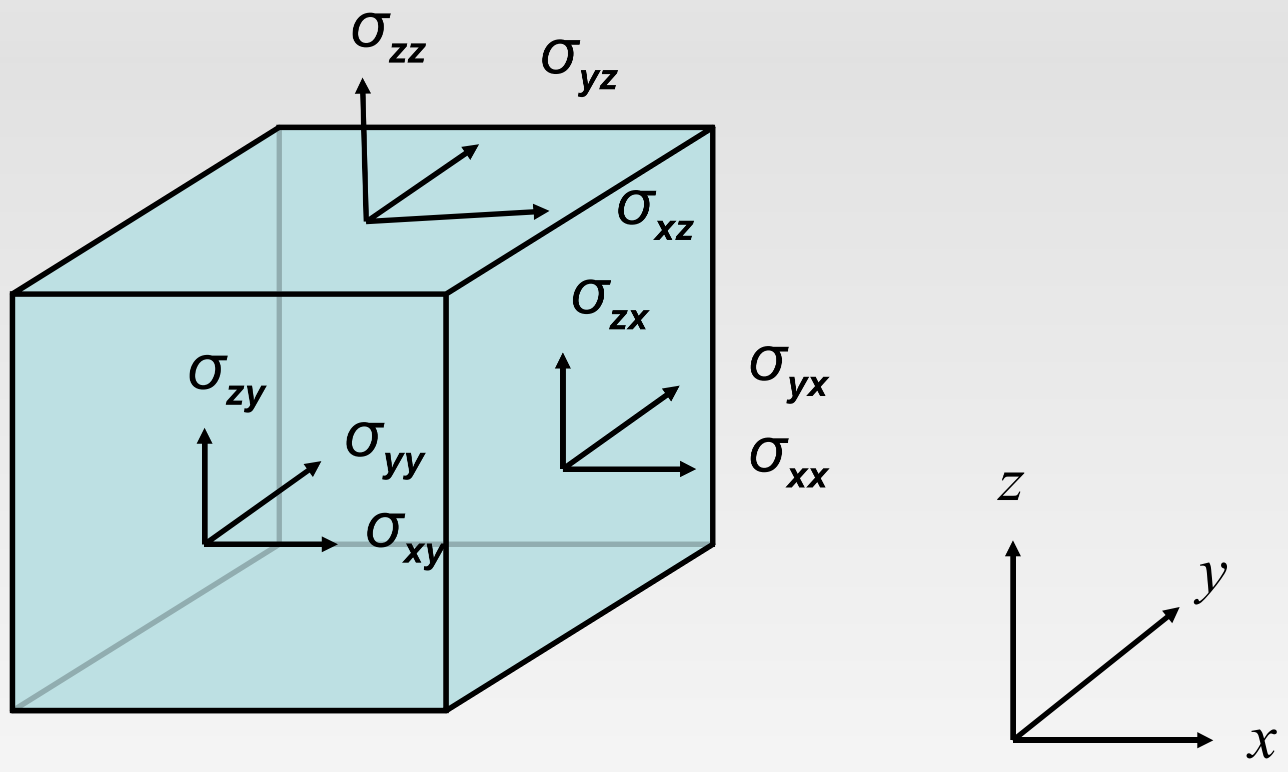
- Flow-line/slice models
  - Flow down a single line in an ice-mass is treated
  - Easier to construct and faster to run than 3D
  - Good for valley glaciers (simple geometry)
  - Good test-beds for models of calving, sliding, etc.
- Further reading:
  - Frank Pattyn's GRANTISM (in Excel)
  - Work by Andreas Vieli, Steve Price, and others

Modelling hierarchy

- Plan-view models
  - Vertically-integrated or three-dimensional
  - Essential for whole ice sheet studies
  - Complex numerical challenges
  - Computationally expensive
- Further reading:
  - Tony Payne, Philippe Huybrechts, Catherine Ritz, Doug Macayeal, and many others

Ice flow: stress balance

Nine balanced stress components



Notation: σ_{ij}

i **the direction in which the stress acts**

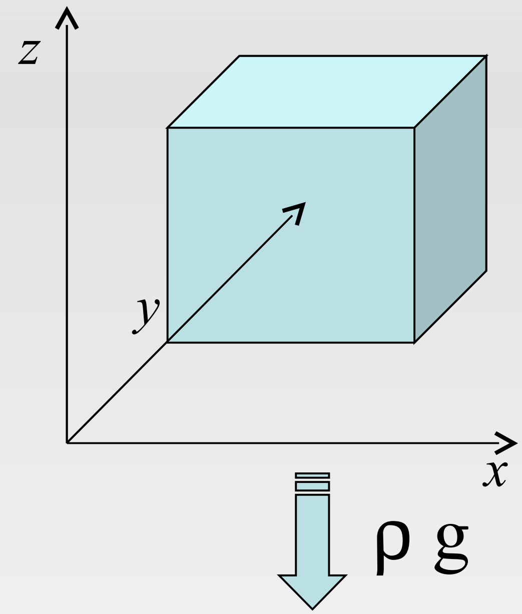
j **the direction normal to the plane on which the stress acts**

Force balance equations

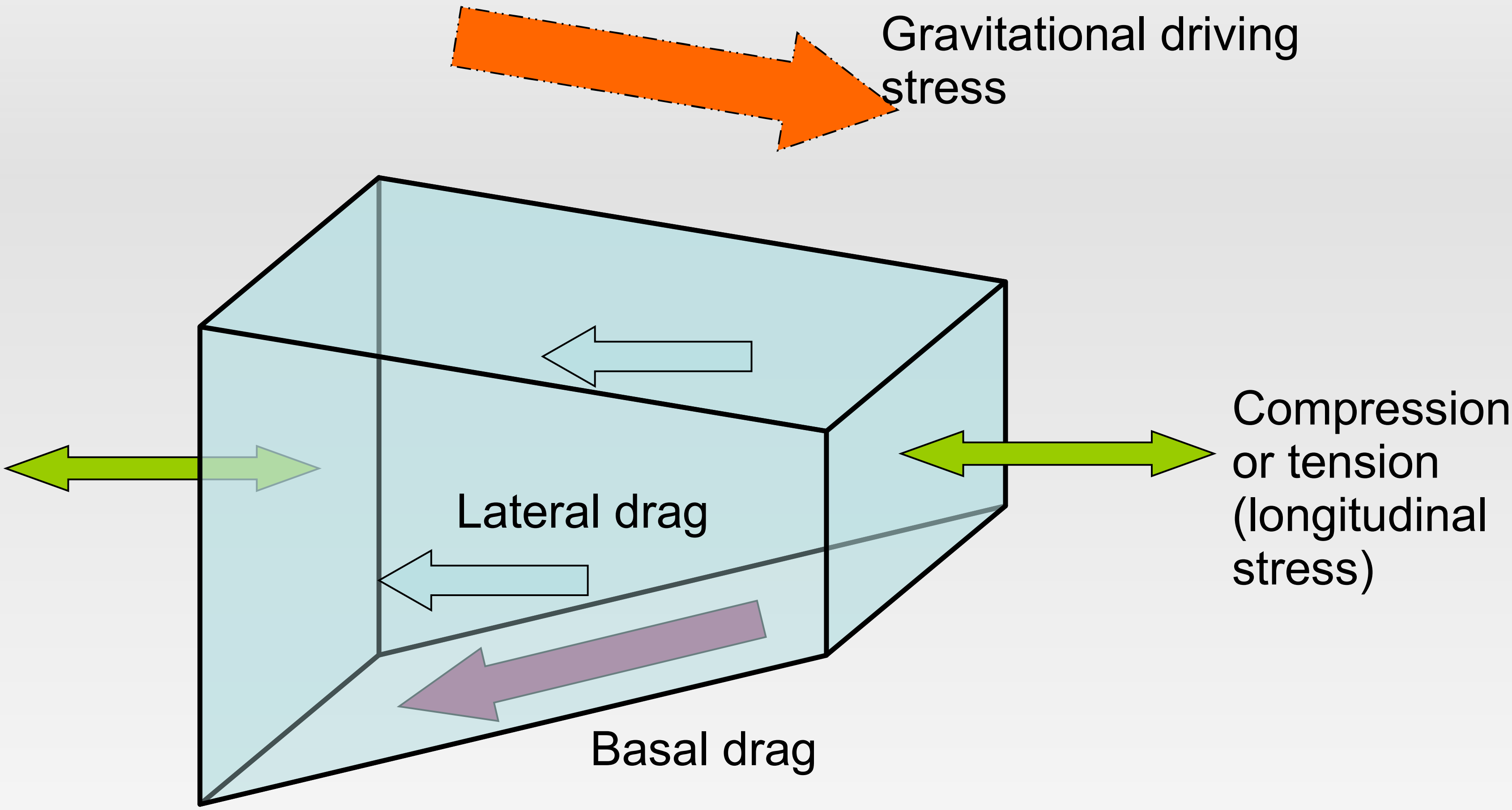
$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} = 0$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial z} = 0$$

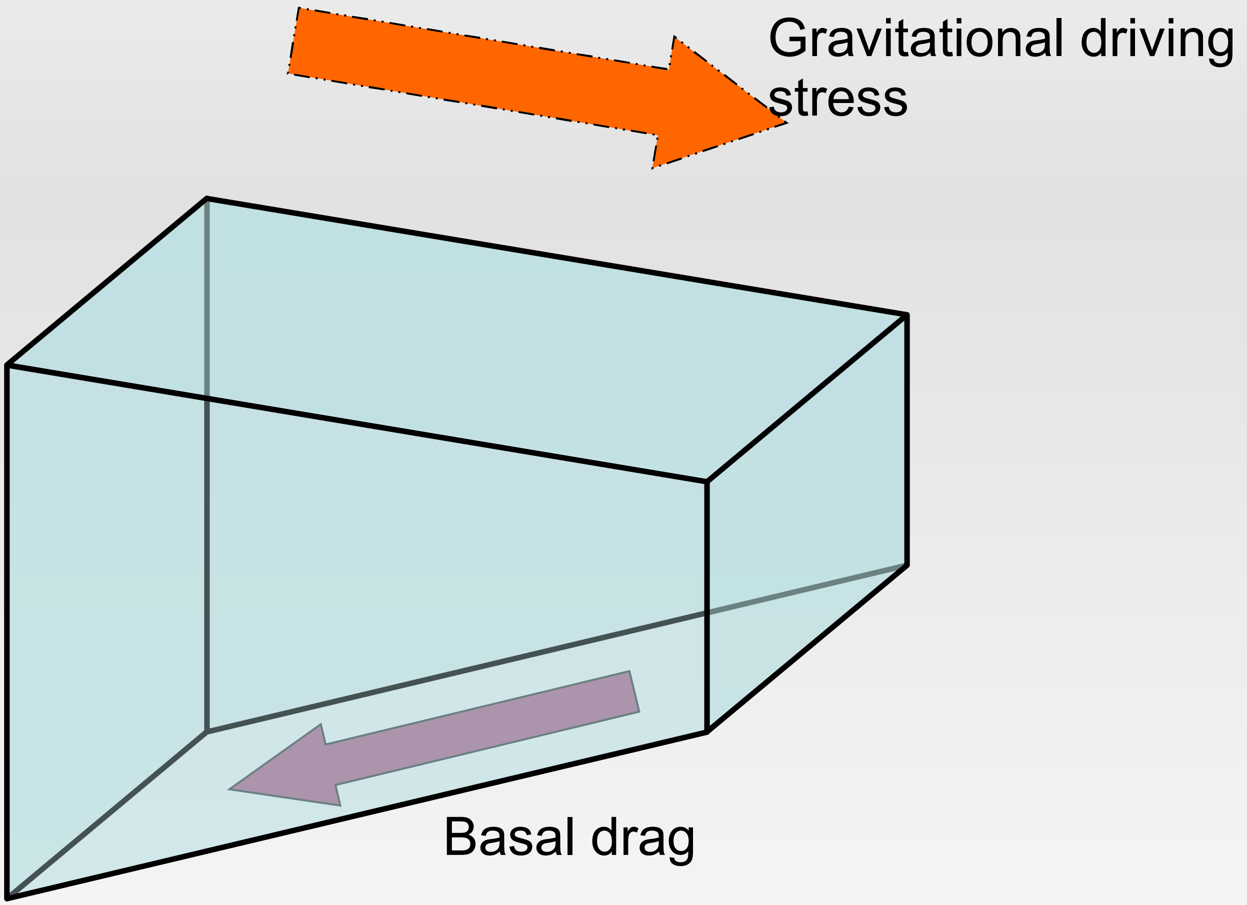
$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} = \rho g$$



Force Balance



Force Balance – Shallow Ice



Shallow Ice Approximation

- assume on basis of aspect ratio that
- normal stress dominates in vertical
- vertical shear stress gradients dominate horizontal shear stress gradients

$$\frac{\partial \sigma_x}{\partial x} + \cancel{\frac{\partial \sigma_{xy}}{\partial y}} + \frac{\partial \sigma_{xz}}{\partial z} = 0$$

$$\frac{\partial \sigma_y}{\partial y} + \cancel{\frac{\partial \sigma_{yx}}{\partial x}} + \frac{\partial \sigma_{yz}}{\partial z} = 0$$

$$\frac{\partial \sigma_z}{\partial z} + \cancel{\frac{\partial \sigma_{zx}}{\partial x}} + \cancel{\frac{\partial \sigma_{zy}}{\partial y}} = \rho g$$

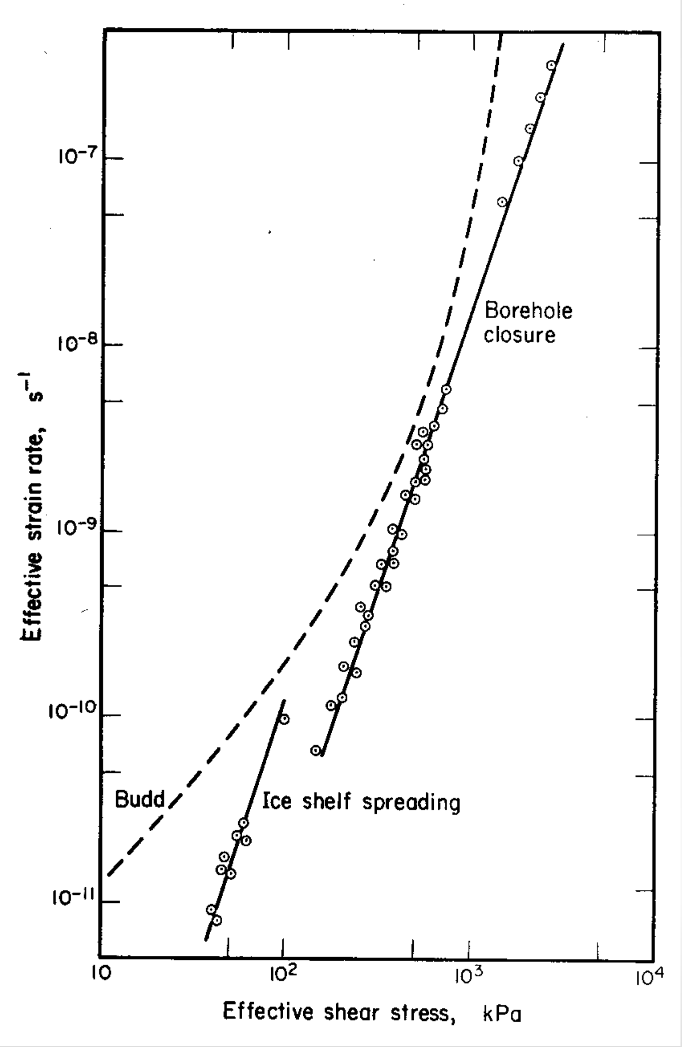
The constitutive relation

Glen's Flow Law is most commonly used

$$\dot{\epsilon} = A\sigma^{n-1}\sigma_{xy}$$

A is temperature-dependent:

$$A = A_0 \exp\left(-\frac{Q}{RT}\right)$$



Shallow Ice – depth integrated

$$\bar{u} = -\frac{2A\left(pg\right)^n}{n+1} H^{n+1} \left|\frac{\partial s}{\partial x}\right|^{n-1} \frac{\partial s}{\partial x}$$

Expression for the local velocity based on known ice sheet geometry (surface gradient and depth) provided...

We ignore several of the stress gradients

Assume vertical gradient of horizontal shear perfectly locally balances horizontal gradient in ice weight.

Assume bed supports all the ice weight locally

The flow law linear constant (A) is uniform with depth

The basis of many models

$$\frac{\partial H}{\partial t} = \frac{-\partial(HU)}{\partial x} + M$$

Time evolution

$$\bar{u} = -\frac{2A(\rho g)^n}{n+1} H^{n+1} \left|\frac{\partial s}{\partial x}\right|^{n-1} \frac{\partial s}{\partial x}$$

Result of stress/
strain balance

Temperature

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C}\frac{\partial^2 T}{\partial z^2} - w\frac{\partial T}{\partial z} - u\frac{\partial T}{\partial x} - v\frac{\partial T}{\partial y} + \frac{\dot{\epsilon}}{\rho C}$$

Heat diffusion

Transport by ice flow

Internal heat generation

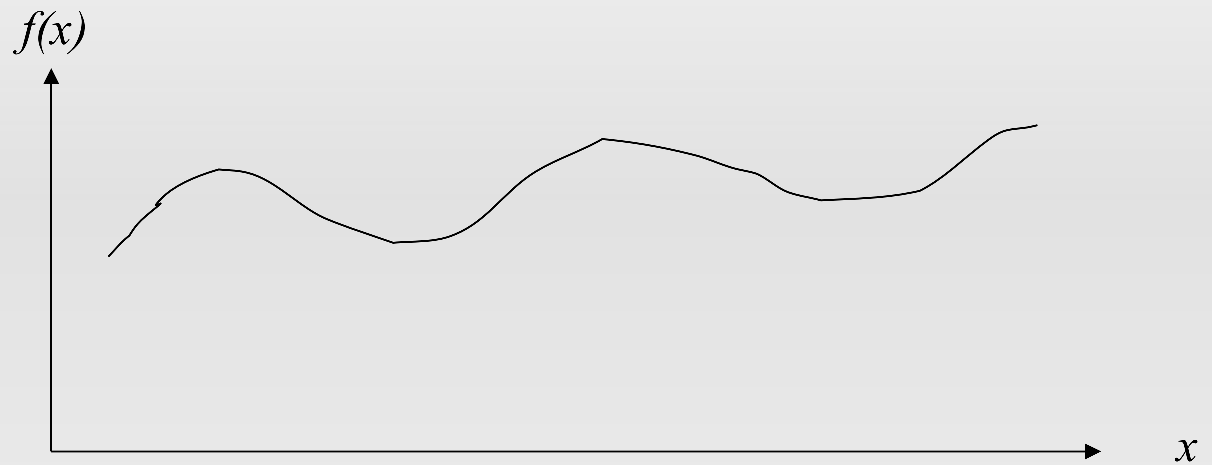
Boundary conditions:

$$T_{z=s} = T_a$$

$$\left.\frac{\partial T}{\partial z}\right|_{z=h} = \frac{-G}{k}$$

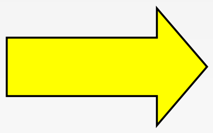
Why do we need numerics?

Continuous function:



…can't be represented on a computer as it is

So, we must divide continuously-varying quantities into discrete chunks so that the computer can handle them.



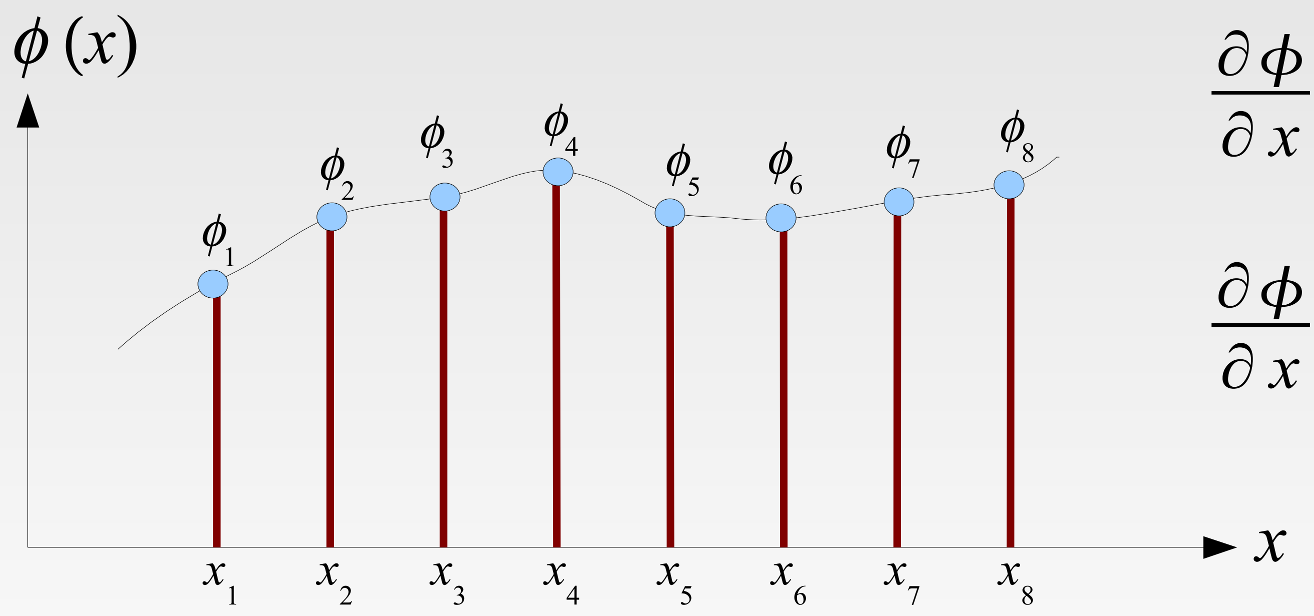
Discretization

Different ways to discretize

- Finite element/finite volume
  - Great freedom in dealing with irregular geometries
  - Fairly complicated to develop although software now available to do many of standard tasks
  - Computationally demanding
  - Notably Jim Fastook and Doug MacAyeal
- Finite difference
  - Simple to use and develop
  - Problems in coping with highly irregular domains

Intuitive explanation

Easy part is to divide a continuous function into a set of discrete points



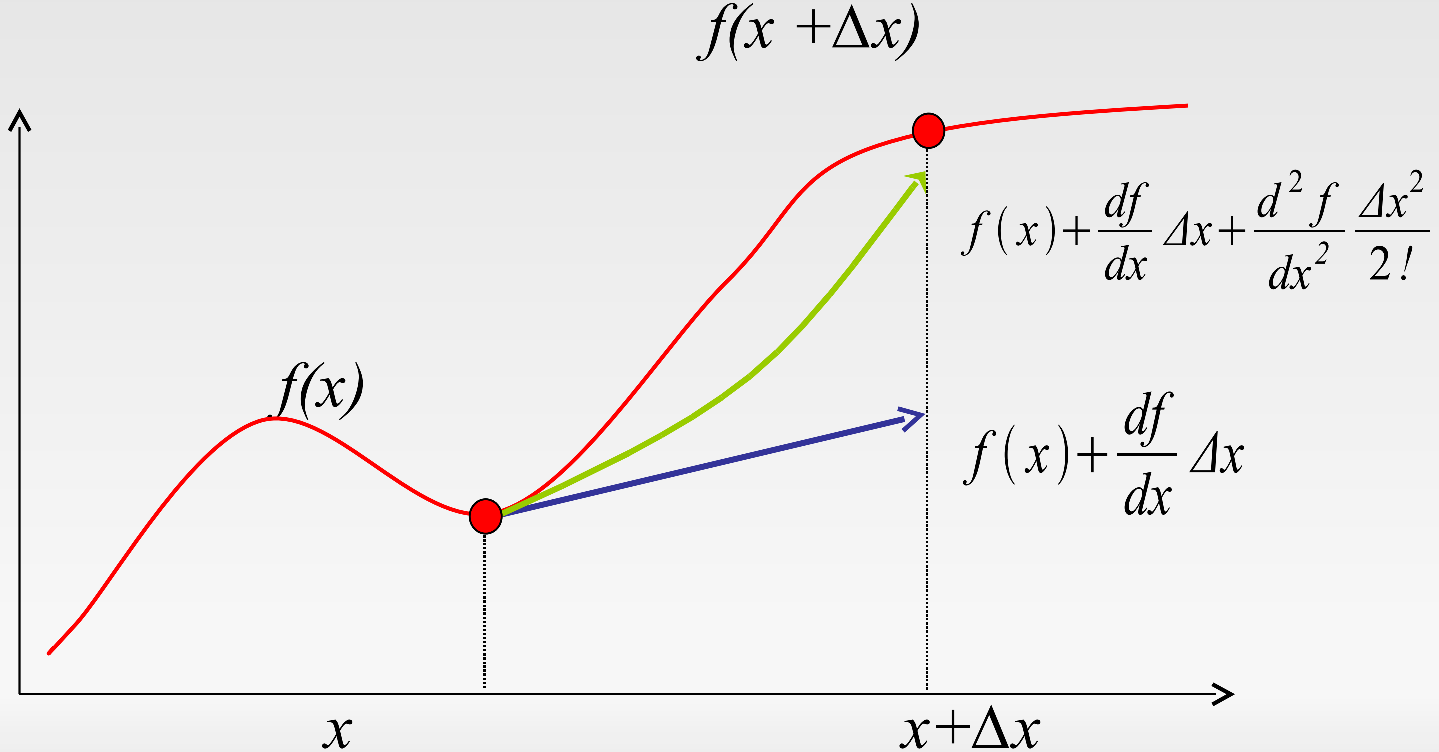
$$\frac{\partial \phi}{\partial x} \approx \frac{\phi_{i+1} - \phi_i}{x_{i+1} - x_i}$$

$$\frac{\partial \phi}{\partial x} \approx \frac{\phi_{i+1} - \phi_i}{\Delta x}$$

Basis of finite differences

Taylor series give a relation between continuous and finite representations

$$f(x+\Delta x)=f(x)+\frac{df}{dx}\Delta x+\frac{d^2 f}{dx^2}\frac{\Delta x^2}{2!}+\frac{d^3 f}{dx^3}\frac{\Delta x^3}{3!}+\dots$$



Basis of finite differences

For convenience, use abbreviated notation:

$$f_{i+1} \equiv f\left(x + \Delta x\right)$$

So, 2nd-order-accurate centred-difference approximation:

$$\left.\frac{df}{dx}\right|_i = \frac{f_{i+1} - f_{i-1}}{2\Delta \mathrm{x}}$$

Similarly:

$$\left.\frac{d^2 f}{dx^2}\right|_i = \frac{f_{i+1} - 2\mathrm{f}_i + f_{i-1}}{\Delta x^2}$$

Advantages of Shallow Ice

- Simple to understand and implement
- Computationally cheap
- Method is well-understood (long history)
- Several established implementations:
  - Glimmer-CISM
  - SICOPOLIS (Greve)
  - Models by Ritz, Huybrechts, etc.

Limitations of shallow ice

- No longitudinal or lateral stresses
  - Doesn't represent ice shelves or ice streams
  - Transition between grounded and floating ice can't be done properly
  - Not appropriate for complex, small-scale changes
- Local stress balance means
  - Basal sliding is difficult to represent properly
  - Zero-stress bed is impossible

Higher-order stress balances

- These represent the stress balance more fully
- Allow representation of shelf and stream flow
- First-order models
  - Pattyn, Blatter, etc.
- Full Stokes models
  - Typically based on general-purpose FE/FV code
  - **Elmer** has been used extensively for Glaciology

Other model components

- Basal sliding
  - In Shallow Ice Model, basal velocity is usually made proportional to driving stress
  - In HO models, more realistic representations are possible
  - May be linked to presence of water, or bed being at pressure melting point
- Basal Hydrology may be important here
  - Very complex area!

Other model components

- Isostatic adjustment
  - Variety of models available
  - Necessary for longer timescales
- Marine margin processes
  - Individual calving events too small to represent in ISM
  - Interaction with sea – under-shelf melt/accretion?

Forcing ice sheet models

- Most important forcings/feedbacks concern the atmosphere
  - Precipitation
  - Melt
  - Refreezing
  - Air temperature
  - Albedo
  - Elevation/roughness

Mass Balance

Climate forcing methods

- Simple integrated schemes (ELA, etc)
- Mass-balance models driven by specified/modelled climate:
  - Degree day methods
  - Energy balance models
- Specifying the climate:
  - Station data
  - Reanalysis data
  - Reduced climate models
  - Fully coupled GCMs

Summary

- Ice Sheet modelling is complex
  - Choice of stress balances
  - Choice of numerical methods
  - Many interacting processes, all of which need modelling
  - Coupling to climate adds another aspect
- BUT:
  - Modelling is going to continue to increase in importance
  - We need more modellers!